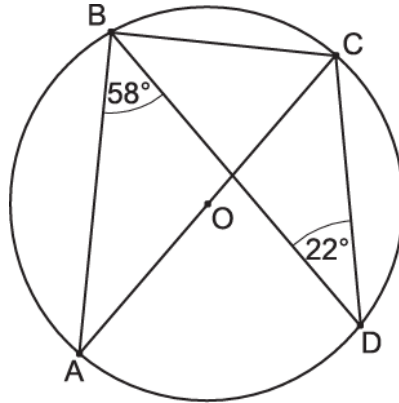


- 1(a). A, B, C and D are points on the circumference of a circle, centre O.  
 AC is a diameter of the circle.  
 Angle ABD =  $58^\circ$ .  
 Angle CDB =  $22^\circ$ .



**Not to scale**

Work out the sizes of angle ACD and ACB, giving reasons for your answers.

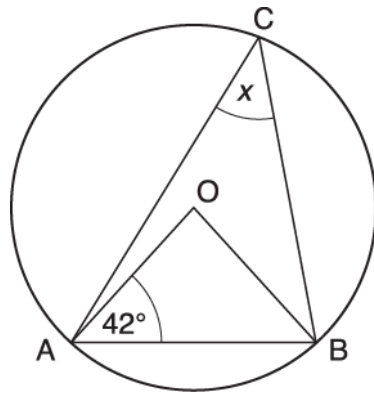
Angle ACD = \_\_\_\_\_ $^\circ$

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 ----- [2]

(b). Angle ACB = \_\_\_\_\_ $^\circ$

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 ----- [3]

2. In the diagram, the points A, B and C lie on the circumference of the circle, centre O.  
Angle BAO =  $42^\circ$ .



**Not to scale**

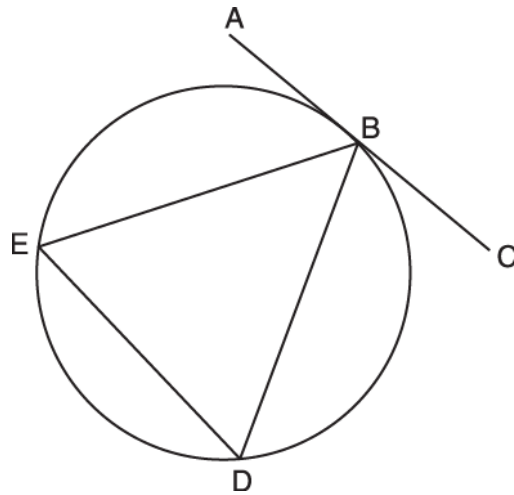
Calculate angle  $x$ .  
Give reasons for each angle you work out.

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[4]



- 3(a). ABC is a tangent to the circle at B.  
D and E are points on the circle.



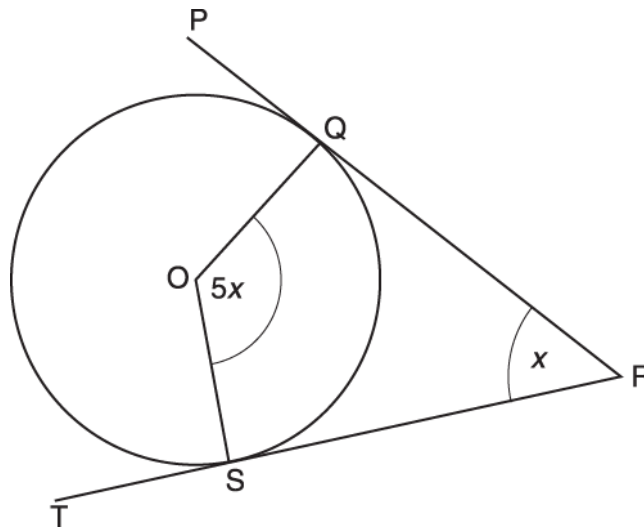
Not to scale

Given that angle CBD is  $81^\circ$ , calculate the size of the **reflex** angle BED.  
Give a geometrical reason for each step of your work.

[3]



- (b). PQR and TSR are tangents to the circle at Q and S.  
O is the centre of the circle.

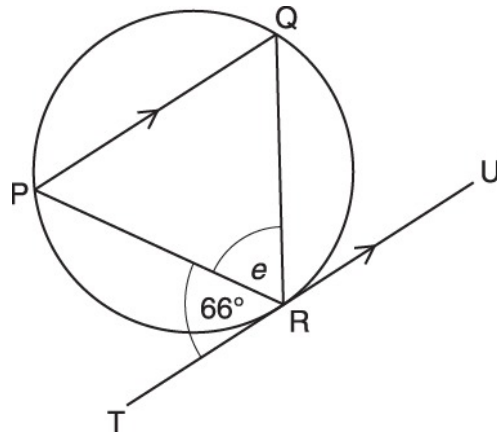


Use algebra to prove that  $x = 30^\circ$ .  
Give a geometrical reason for each step of your work.

[4]



4. Chord PQ is parallel to tangent TRU.



Not to scale

Calculate the size of angle  $e$ .

Give a geometrical reason for each stage of your working.

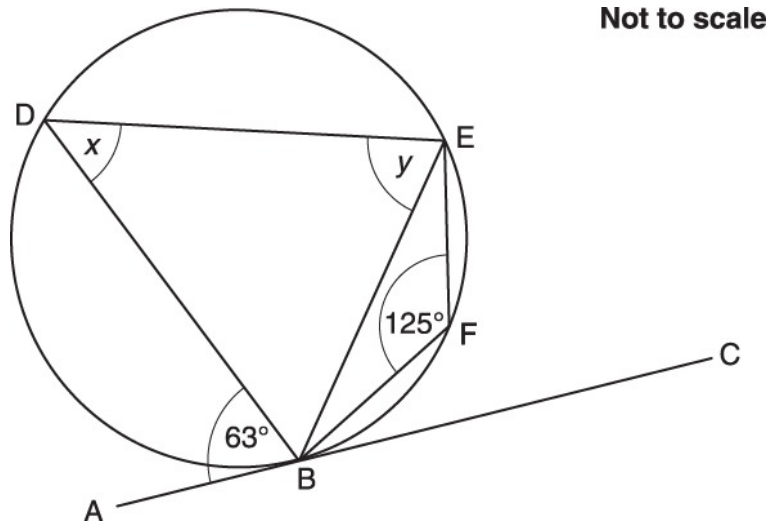
[5]



5(a). In the diagram, BDEF are points on the circumference of a circle.

AC is the tangent to the circle at B.

Angle ABD =  $63^\circ$  and angle EFB =  $125^\circ$ .



Find each of these angles, giving a reason for your answers.

x = \_\_\_\_\_  $^\circ$  because \_\_\_\_\_

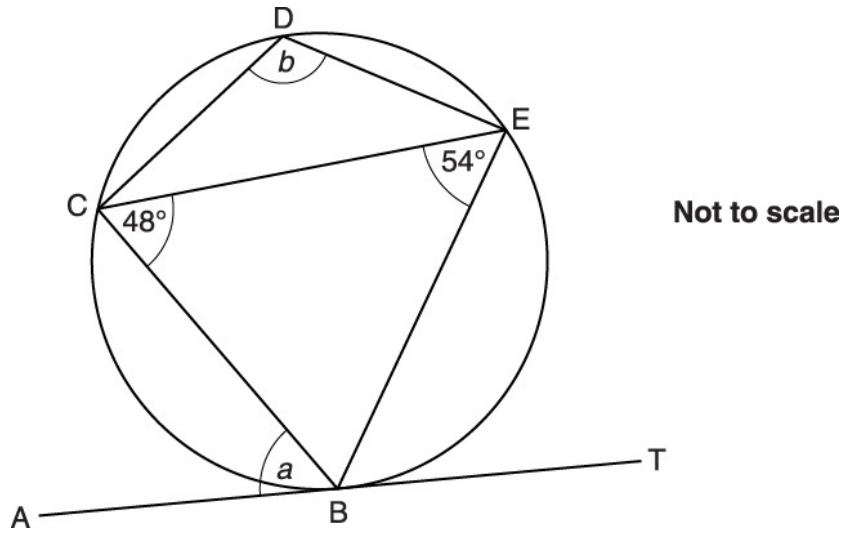
----- [2]



(b). y = \_\_\_\_\_  $^\circ$  because \_\_\_\_\_

----- [2]

- 6(a). In the diagram B, C, D and E are points on the circumference of a circle.  
 AT is the tangent to the circle at B.  
 Angle BCE =  $48^\circ$  and angle BEC =  $54^\circ$ .



Find angle  $a$ .  
 Give a reason for your answer.

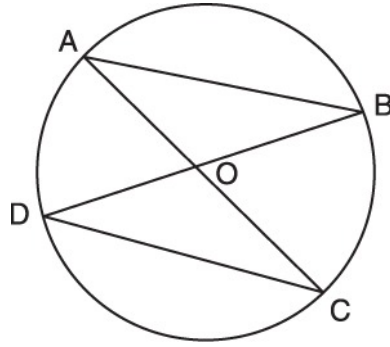
Angle  $a =$  \_\_\_\_\_  $^\circ$

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 ----- [2]

- (b). Calculate angle  $b$ .  
 Give a reason for each step of your working.

Angle  $b =$  \_\_\_\_\_  $^\circ$  [3]

7. In the diagram AC and BD are diameters of the circle, centre O.



**Not to scale**

Prove that triangles OAB and ODC are congruent.

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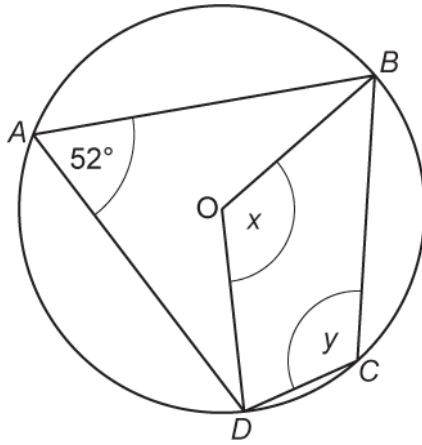
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**[3]**



8(a). A, B, C and D are points on the circumference of a circle, centre O.



Not to scale

Angle BAD =  $52^\circ$ .

Work out angle  $y$ .

Give a reason for your answer.

$y =$  .....  $^\circ$  reason .....

..... [2]



(b). Work out angle  $x$ .

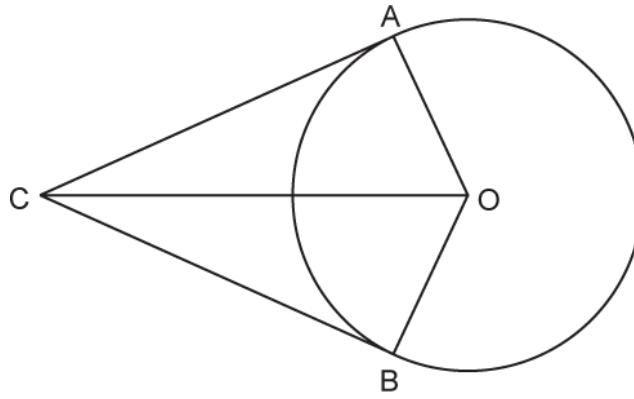
Give a reason for your answer.

$x =$  .....  $^\circ$  reason .....

..... [2]



9. A and B are points on the circumference of a circle, centre O.  
CA and CB are tangents to the circle.



Not to scale

Prove that triangle OAC is congruent to triangle OBC.

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[4]

END OF QUESTION PAPER

Question		Answer/Indicative content	Marks	Part marks and guidance	
1	a	58° Subtended on same arc oe	2	B1 for angle	
	b	68° e.g. angle DBC is 32° because the angle in a semicircle is a right angle oe so angle ACB is 68° because angles in a triangle sum to 180° oe	3	B1 for using the angle in a semicircle is a right angle B1 for using angles in a triangle sum to 180°	
		<b>Total</b>	<b>5</b>		
2		angle ABO = 42° or angle AOB = 96°  'angles [in a] triangle [add up to 180]' or 180 [in a] triangle or isosceles [triangle]  [x = ] 48	B1  B1  B1	may be on diagram and implied by the correct answer   may be on diagram	Note: the correct answer scores B2

Question			Answer/Indicative content	Marks	Part marks and guidance
			angle [at the] centre is twice [the angle at the] circumference' oe	B1	<p>or angle [at the] circumference is half [the angle at] the centre</p> <p><b>Examiner's Comments</b></p> <p>Some of the reasons given were wrong, and some were completely omitted, but the value of <math>x</math> as 48 was usually correct. The two most common errors were those who either thought the angle AOB, or the angle ABC, was 90.</p> <p>There were also those who used BAC as 42, even when it was quite clearly only BAO. The reason 'angles in a triangle add to 180' or 'isosceles triangle' were usually present, whilst 'angle at the centre is twice that on the circumference' was only sometimes present. It should be noted that 'edge' is not sufficient for 'circumference' nor is 'middle' or 'origin' for 'centre'. There were many with the correct answer but gave no reasons.</p>
			<b>Total</b>	<b>4</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
3	a	279  Alternate segment [Angles round a point = 360]	2  1	<b>B1</b> for BED = 81 or 360 – <i>their</i> BED	If parallel lines etc assumed the reason mark is not earned  <b>Examiner's Comments</b>  Many candidates found BED as 81° although some did it by first assuming that AC and ED were parallel. The most able candidates, however, could concisely and confidently use and explain the alternate segment theorem. Many candidates either did not notice the demand for the reflex angle, despite it being in bold type, or more worryingly did not understand what a reflex angle is.
	b	Valid geometric reason  Valid geometric reason  Valid algebraic approach eg to angles in triangle, quad'l etc	B1  B1  M1	To a max 2	eg Angle between tangent and radius = 90° Angles in a quad'l = 360 eg $x + 5x + 90 + 90 = 360$

Question			Answer/Indicative content	Marks	Part marks and guidance
			Clear proof that hangs together eg all triangles/angles defined	A1	<p><b>Examiner's Comments</b></p> <p>Only the most able candidates could understand the demand for a proof. They stated in full the reasons they were using and then showed clear, concise algebra to finish up with <math>x = 30^\circ</math>. Less able candidates often scored a mark or two for quoting a relevant reason but others started off by assuming <math>x = 30^\circ</math> thus making it hard for them to earn any marks.</p>
			<b>Total</b>	<b>7</b>	

Question		Answer/Indicative content	Marks	Part marks and guidance	
4		* Correct angles with correct working and reasons, clearly laid out	5		$\angle RPQ = 66^\circ$ alternate angles $\angle PQR = 66^\circ$ alt seg $e = 180 - 66 - 66$ $= 48^\circ$ angles in a triangle
		As 5 but one missing reason Or full solution with no more than 1 arithmetic error	4-3	For the lower mark the answer will be correct with > 1 missing reasons	$\angle RPQ = 66^\circ$ alternate angles $\angle QRU = 66^\circ$ alt seg $e = 180 - 66 - 66$ $= 48^\circ$ angles on a straight line Condone Z for alternate angles
		2 correct angles Or one correct angle with reason	2-1	For the lower mark there will be one correct angle	Accept angles marked on diagram for 1 or 2 marks
		No correct work seen		<b>Examiner's Comments</b>  This question assessed candidates' Quality of Written Communication (QWC), so they were expected to give reasons for their conclusions. Many candidates scored full marks usually for finding RPQ first often quoting 'alternate angles' rather than the less precise 'Z-angles', then using the alternate segment theorem to give PQR and finally angles in a triangle. Most candidates got as far as the correct answer for e but only the strongest were able to quote 'alternate segment' correctly. Some candidates gave long explanations of this theorem often with extra diagrams but they did not get the final mark without quoting 'alternate segment' (although the abbreviation 'alt seg' was condoned).	

Question			Answer/Indicative content	Marks	Part marks and guidance
			Total	5	

Question		Answer/Indicative content	Marks	Part marks and guidance	
5	a	55  opposite [angles of a] cyclic quadrilateral [add to 180] oe	1  1	<p><b><u>Examiner's Comments</u></b></p> <p>There was evidence of poor understanding of cyclic quadrilaterals. When the answer 55 was given the reason was often omitted or they used just the words "opposite" or "cyclic". Some even stated that "opposite sides added to 180". A common answer seen was 62.5 from half of 125 with "the angle at the centre", other incorrect reasons seen included "63 with corresponding or alternate angles", thinking AC and DE were parallel, "angles in a four-sided shape add to 360", and "angles in a triangle add to 180".</p>	
	b	63  alternate segment [theorem]	1  1	<p><b><u>Examiner's Comments</u></b></p> <p>Candidates struggled to work out the answer here clearly indicating that the "alternate segment theorem" is not well known. Those who had already put 63 for part (a) were at a loss how to proceed and many struggled to use the language required to describe their reasoning. Incorrect reasons given included "alternate angles", "corresponding angles", "angles subtended from the same chord are equal" and "angles in the same segment are equal".</p>	
		<b>Total</b>	<b>4</b>		



Question		Answer/Indicative content	Marks	Part marks and guidance	
6	a	54	1	<p><b>Examiner's Comments</b></p> <p>The alternate segment theorem was not well known and therefore this question was answered poorly. Many thought that the answer was 48 because of alternate angles, yet the two lines are not drawn parallel, nor is there any indication that they are parallel.</p>	may be on diagram
		alternate segment [theorem]	1		
	b	102	1	<p><b>Examiner's Comments</b></p> <p>There was a lack of correct reasons, particularly "the opposite angles of a cyclic quadrilateral add up to 180". There was no need to use the four angles of the quadrilateral, as the easiest method would have been to work out angle CBE first using the triangle, then use the property of the opposite angles.</p>	may be on diagram
angles [in a] triangle [add to 180°] or triangle [adds to] 180° or angle[s on a] straight line		1			
		opposite and/or angles [in a] cyclic quadrilateral [add up to 180°]	1		
		<b>Total</b>	<b>5</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
7		States $\angle AOB = \angle DOC$ and $AO = DO$ and $BO = CO$	B1	Or two pairs of angles and one pair of sides $\angle OAB = \angle ODC$ , $\angle ABO = \angle DCO$	Condone $AO = CO$ and $BO = DO$ Allow angles named A, B, C, D but $\angle AOB$ and $\angle DOC$ must be clearly identified
		States a correct reason for a pair of angles and a correct reason for a pair of sides [vertically] opposite [angles] [equal] radii	B1	Or [angles] same segment [equal] or [angles on] same chord/arc [equal]	Condone 'half diameter' for radius
		Selects correct congruence statement for their argument	B1	SAS or ASA  After B0 award SC1 for two correct pairs of sides and/or angles with correct reasons seen  <b>Examiner's Comments</b>  Almost all candidates performed very badly on this question, with only a small proportion gaining any marks at all.  A proof requires clear statements giving equal sides and equal angles with correct geometrical reasons concluding with a correct congruence statement. Angles were often paired correctly, but reasons for these were often incorrect or omitted, in particular for the equal angles in the same segment. In some cases sector was used in place of segment, but, more commonly the 'bow tie theorem' was referred to, which is not an acceptable reason. It was often assumed that AB and DC were parallel and 'alternate angles' was used which was not accepted. If the	Or AAS

Question			Answer/Indicative content	Marks	Part marks and guidance	
					<p>radii were paired up, a reason for this was seldom adequate, with pairs of diameters often mentioned rather than radii. Those candidates who had correctly paired angles and sides did not often then go on to give a correct congruence statement.</p> <p>A number of candidates confused congruence with similarity and attempted to prove that the angles in the two triangles were equal with no mention of equal sides.</p>	
			<b>Total</b>	<b>3</b>		

Question		Answer/Indicative content	Marks	Part marks and guidance	
8	a	128 <u>Opposite angles</u> in a <u>cyclic quadrilateral</u> [are supplementary oe]	2	B1 for 128	<p>With no incorrect statement Must use underlined terms Condone opp angles in cyclic quad = 180</p> <p><b>Examiner's Comments</b></p> <p>There were fewer correct angles in part (b). A value of 76 was relatively common as a wrong answer. Where the angle was correct the second mark could often not be given because the term 'cyclic quadrilateral' was not used; candidates often gave more general descriptions, such as 'quadrilateral in a circle'.</p>

Question		Answer/Indicative content	Marks	Part marks and guidance	
	b	104 <u>Angle at centre is twice</u> angle at <u>circumference</u>	2	B1 for 104	<p>With no incorrect statement Must use underlined terms. Accept reverse: <u>angle at circumference</u> is <u>half</u> angle at <u>centre</u> Accept arc for circumference but not edge</p> <p><b>Examiner's Comments</b></p> <p>Most calculated the value of the angles correctly, but fewer were able to give the correct geometric reasons leading to these values using appropriate mathematical terminology. In part (a), some candidates simply said one angle was double the other. Other attempts at a correct reason were often inadequate with the use of words such as 'origin' instead of 'centre' or 'edge' instead of 'circumference'.</p>
		Total	4		

Question		Answer/Indicative content	Marks	Part marks and guidance		
9		<p>Three of these</p> <ul style="list-style-type: none"> <li>• OC is common or shared</li> <li>• OA = OB (equal) radii</li> <li>• <math>\angle OAC = \angle OBC</math> tangent perpendicular to radius</li> <li>• CA = CB tangents from a point [to a circle]</li> </ul> <p>RHS, SSS or SAS as appropriate</p>	<p><b>M3</b></p> <p><b>A1</b> 4 AO2.4b</p>	<p>M1 for each</p> <p>After M0, B2 three pairs of these equal sides / angles with insufficient or no reasons</p> <p>or</p> <p>B1 for two pairs of these equal sides / angles identified with insufficient or no reasons</p> <p>OR</p> <p>After M1, B1 for two further pairs of these equal sides / angles identified with insufficient or no reasons</p>	<p>Ignore extra facts and reasons</p> <p>For B marks accept if indicated on diagram</p>	<p><b>Examiner's Comment</b> Full marks were awarded only occasionally and these responses were usually characterised by their clarity and brevity. Many used an essay style with the commentary not linked to specific sides or angles. In particular, for AC = BC many candidates omitted to</p>

Question			Answer/Indicative content	Marks	Part marks and guidance
					<p>mention that these tangents met at a point. When dealing with angles <math>OAC = OBC</math>, many omitted either <math>90^\circ</math>, tangent or radius (for example 'The tangents meet the circumference at <math>90^\circ</math>' was frequently seen). Although the award of full marks was rare, many obtained 2 marks for giving pairs of equal angles / sides without sufficient reasons.</p>
			<b>Total</b>	<b>4</b>	